

Static analysis and evaluation of the construction system of a gothic «choir-window» consisting of a filigree tracery and slender stone ribs

R. Barthel
L. Schiemann
M. Jagfeld

The St. Georg church at Nördlingen located in the south of Germany shows a large gothic tracery window with a height of more than 12 metres. Damages at the stone bars and the tracery are the reason for an assessment of the structure and a statical analysis. The framework consists of sandstone and wrought-iron saddle bars. The stone bars are extremely slender. The analysis is carried out by means of the finite element method, considering the opening of the joints under wind load. A theoretical model of the principle statical behaviour is developed taking into account the sequence of construction and the stiffness of the structural elements.

INTRODUCTION

St. Georg at Nördlingen, located near Augsburg in the south of Germany, is one of the last churches of the late gothic period. Its significance is not only due to its great dimensions in height and length but also to its accurate technical construction. The height of the nave is 18.6m, the length is 93.5 m, the width 23.5 m. The pillars are extremely slender. The precise stonework of the large filigree gothic tracery also emphasizes the high quality of the building (fig. 1). The construction of the building was begun in 1427 at the choir and was completed already after 92 years in 1519.



Figure 1
St. Georg at Nördlingen with the tracery window in the
middle

THE CHOIR WINDOW

Geometry and materials

The height of the window is nearly 12.30 metres, the width is 3.70 metres (fig. 2). At a level of approximately 12.0 metres a typical stump tracery (fig. 2, pos. 6) is constructed.

The framework consists of five slender stone bars - two main bars (fig. 2 pos. 1) and three minor bars (fig. 2, pos. 2) - each of them 7.85 m long. The five stone bars divide the glass front of the window into six stripes. Bars consisting of wrought iron (fig. 2, pos. 3), span from one side of the window to the other, piercing the stone bars at their joints.

At about half of the height a horizontal beam interrupts the over all framework (fig. 2, pos. 4). It provides an additional stiffness in horizontal

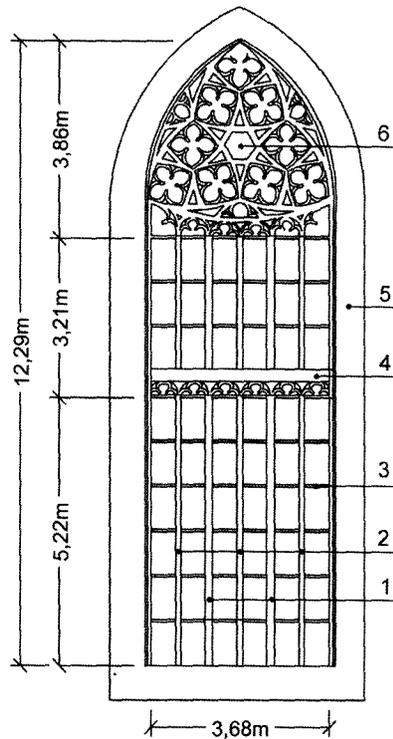


Figure 2
Choir window: 1 main bars 2 minor bars 3 saddle bars 4 central beam 5 reveal 6 tracery

direction. The assessment of the connection between the horizontal beam at the reveal proves, that additional stiffening was considered as necessary during construction time (fig. 6). It was not planned in advance.

The framework including the tracery consists of two different cross sections. The big one is about 15 cm wide and 30 cm deep, the small one is about 12 cm wide and 23 cm deep (fig. 3, 4). The horizontal beam consists of a tracery at the bottom and a full cross section at the top. It is 30 cm deep and 58 cm high including the tracery.

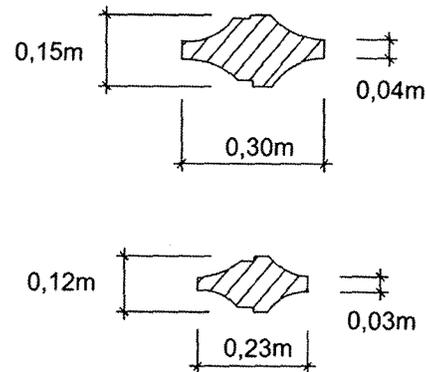


Figure 3
Cross-sections, top: main bar, bottom: minor bar

The material of the framework of the tracery and of the horizontal beam is a local sandstone.

In a course of tests at test pieces the following 'material constants' for the sandstone were determined:

- average compression strength: $\beta_m = 40.65 \text{ N/mm}^2$
- average density: $\rho_m = 2.15 \text{ g/cm}^3$

The wrought-iron saddle bars are conducted as rectangular cross sections (30x50mm) and have after the tests the following material indices:

- average tensile yield strength: $f_{y,k} = 247.2 \text{ N/mm}^2$
- average ultimate strength: $f_{u,k} = 347.7 \text{ N/mm}^2$

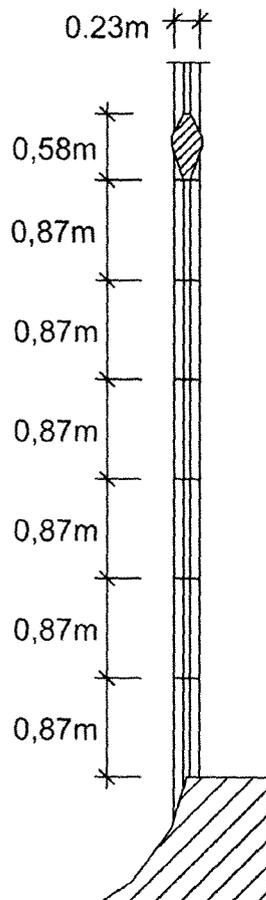


Figure 4
View of minor bar with joints (drawing true to scale)

The reveal (fig. 2, pos. 5) is made out of the very resistant local stone called suevit.

Figure 5 shows the structural joints and the segments of the tracery. The originally large sized individual segments of the tracery are a particular characteristic of the choir window.

Damages

The assessment of damages shows:

- some bars of the framework are not anymore precisely vertical,

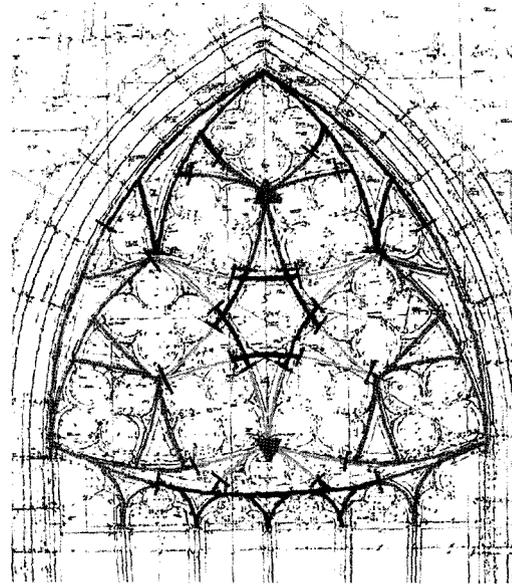


Figure 5
Cutting of stones of the tracery and structural joints

- at the connection between the horizontal beam and the reveal gaps and cracks occur (fig. 6).
- at the anchoring points of the saddle bars the stone is broken (fig. 7),
- originally joints of the tracery show gaps of about two centimetres width (fig. 8),
- some of the stones segments of the tracery are broken.

The main reason of the damages are horizontal movements of the lateral walls adjacent to the window. A deformation of the window arch or an enlargement of the width leads inevitably to disturbances of the structure of the tracery. Such movements can have different reasons. The horizontal thrust of a damaged roof structure or the thrust of the choir vaults, settlements of the foundation and vibrations due to an explosion in the second world war may have caused the movements.

STATIC BEHAVIOUR

It is a very common assumption that the wrought-iron saddle bars take the wind load. That seems reasonable

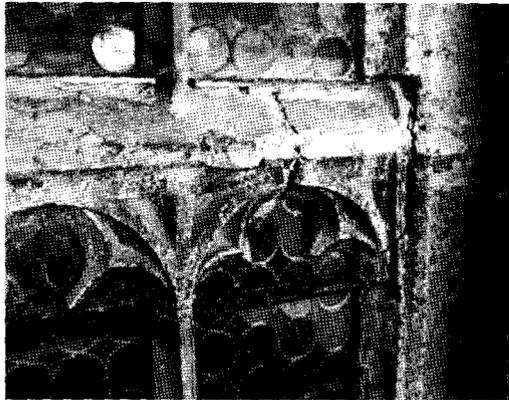


Figure 6
Support of the horizontal central beam

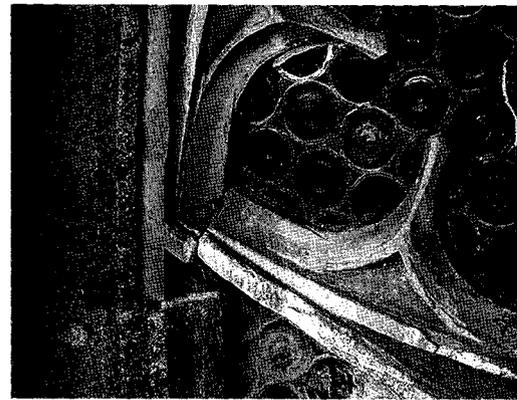


Figure 8
Broken joint at the tracery

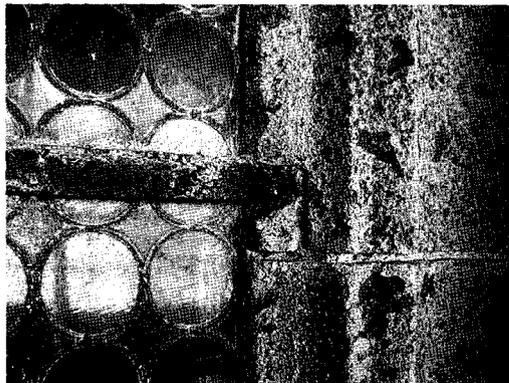


Figure 7
Anchoring of saddle bar

because of the extreme slenderness of the stone bars. The statical analysis of the saddle bars lead to a different conclusion.

Saddle bars

Assuming that the complete wind load is taken only by the saddle bars the stress due to the bending moment is 19 kN/cm^2 . This value could be acceptable. However the deflection is very big. The maximum displacement is nearly 5.0 cm . It is approximately $1/73$ of the span, far beyond $1/300$ of the span, which is considered as reasonable for

modern structures. However the deciding fact is, that such a displacement is far too big for the stone framework. The stonework is much stiffer. Big damages like crushing and spalling of the edges at the joints would occur. The assessment of damages do not show such severe damages. The conclusion is that the stone skeleton takes the main part of the wind load.

The stone skeleton

In the following, it is assumed, that the wind load is taken completely by the stone skeleton.

The statical analysis of the framework work is carried out using the finite element program ANSYS. The material model is linear elastic. The modulus of elasticity is assumed to 5000 kN/m^2 . The opening of the joints is taken into account by an iterative method. The stiffness of the elements at the joints which show tensile stress are reduced to nearly zero. Then the calculation is repeated until the results show no significant tensile stress anymore. The analysis is based on the deflection theory. The iterative solution method takes into consideration the initial imperfections and big deformations. In order to determine a safety factor the wind load has to be increased up to the failure of the structure. It can occur due to material failure or due to snapping through. One bar is modelled using about 5000 solid elements. These elements are normally used for the

three-dimensional modelling of solid structures. They are defined by eight nodes having three degrees of freedom at each node (fig. 9). The simulation of the joints require the big amount of elements. The results, especially the reaction at the supports, are checked by comparative analysis methods as far as possible.

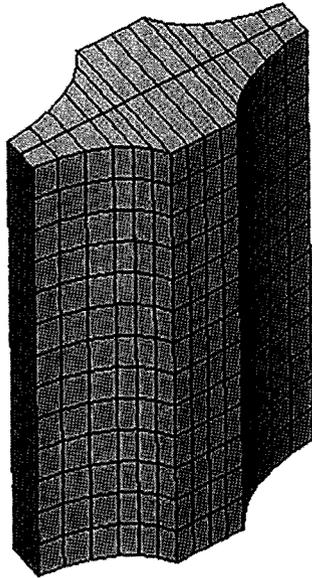


Figure 9
Modelling of a minor bar with three-dimensional elements

The overall structural framework can be subdivided into four separate substructures:

- the vertical bars below the horizontal beam
- the vertical bars above the horizontal beam
- the horizontal beam and
- the tracery at the top.

The slender bars of the framework require an axial compression force to be able to transfer wind load. Only in this case the bars can take bending moments. The axial forces can result from self weight and by creating a flat arch action between rigid supports. The rigid supports can be provided by the heavy masonry around the window. It has to be discussed later how

to ensure the stiffness of these supports. In the following it is assumed that they are absolutely rigid.

Concerning the self weight it is necessary to distinguish two cases. If the tracery at the top is supported only by the vertical bars the maximum of the axial force is acting. If a significant part of the self weight of the tracery is supported by friction at the connection to the reveal the axial force is smaller. In this case the framework is stressed only by the self weight of the bars and of the horizontal beam (fig. 10). This is the worse case in view of the additional wind load.

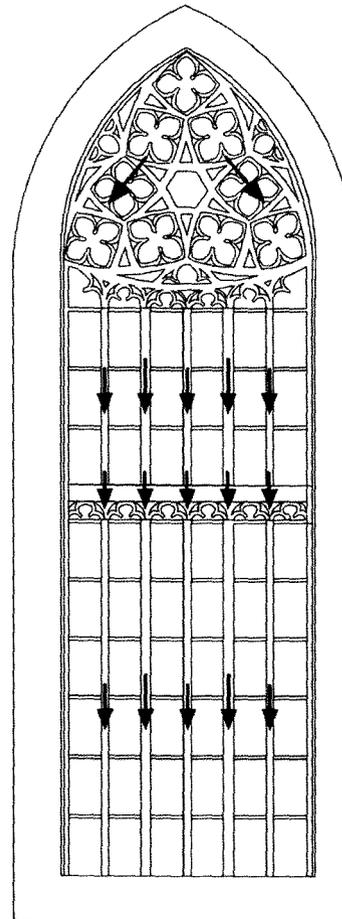


Figure 10
Load case minimal self weight

The analysis shows, that the self weight is not high enough to enable the vertical bars to take the full wind pressure or wind suction. A bar which is fixed at the bottom and supported only horizontally at the top is not stable under wind load. In addition to the axial forces due to self weight the forces due to the arch action have to be taken into account in order to get a state of equilibrium.

The structural model of the window is illustrated in fig. 11. It is assumed that the horizontal beam is acting as a rigid support for the bars in horizontal and vertical direction. The stiffness in horizontal direction is provided by creating a flat horizontal arch inside the beam. Therefore the masonry on both sides of the window has to be stiff enough.

The stiffness in vertical direction is provided by the heavy masonry upon and underneath the window. Regarding the vertical forces it is possible to assume a diaphragm action at the tracery. This assumption has to be proved separately.

The model works only on condition that all joints are very stiff and no gaps or cracks interrupt the flow of forces. The shear forces also have to be transmitted at the joints. Both preconditions are not unrealistic.

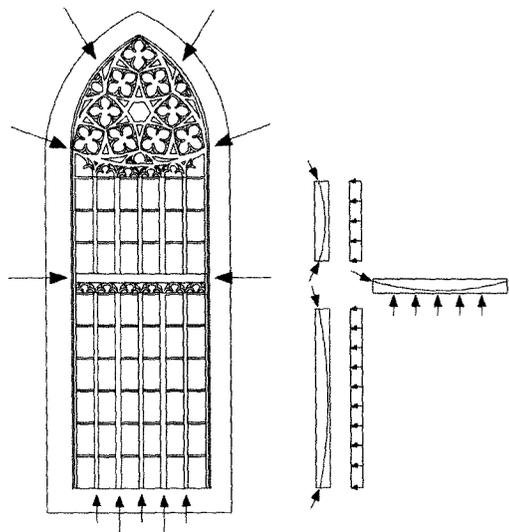


Figure 11
Structural model. Left: supporting reactions due to wind load. Right: vertical bars and horizontal beam subjected to wind load (principal diagram)

The assessment of the window shows that the joints between the stones are very thin even in cases their lead is used as fill. At the tracery there are dowels inside the joint to ensure a shear stiffness.

Statical analysis of a minor bar

In the following the analysis of the most slender bar below the horizontal beam is described and some results are given to serve as an example. Defining the static system and its boundary conditions the sequence of construction has to be taken into consideration. The framework of the window was built after the walls of the choir have been finished. Setting up the stones the bars get under compression and suffer an elastic deformation. Closing the last joint at the top of the window a situation is created which can be considered as a pre-stressed structure between rigid boundaries.

This situation has to be simulated in two steps using two different static systems (fig. 12). In the first step the elastic shortening (u_z) due to self weight (G_N) and due to the weight of the elements above (G_1) has to be determined. Therefore the first system is fixed at the bottom and only horizontally fixed at the top (fig. 12a). The second system has rigid supports at the bottom and the top. In this system tensile stress is created at the top due to the self weight. Now the vertical displacement u_z which was determined in the first step has to be imposed to the system. The result is a system which simulates the real situation between the rigid boundaries (fig. 12b). It is the starting point for subjecting the bar to wind load. The iterative analysis described above now has to be carried out.

The most important result is that a state of equilibrium under wind load is possible. The reaction forces in vertical direction due to self weight plus wind load are significantly higher than due to self weight only. The flat arch action is an essential feature of the static behaviour. The joint in the middle open up to the half of the cross section. The stress distribution at the joints are given in figure 13. The maximum deflection is 0.5 cm.

In figure 12 and 13 the results are given for maximum self weight. In case of minimal self weight the reaction forces are smaller ($G_N = 1.77$ kN, $G_1 = 2.88$ kN, $V_{G_1} = 4.65$ kN, $V_{G+w,u} = 9.39$ kN, $V_{G+w,o} = 5.84$ kN). However the compressive stress is

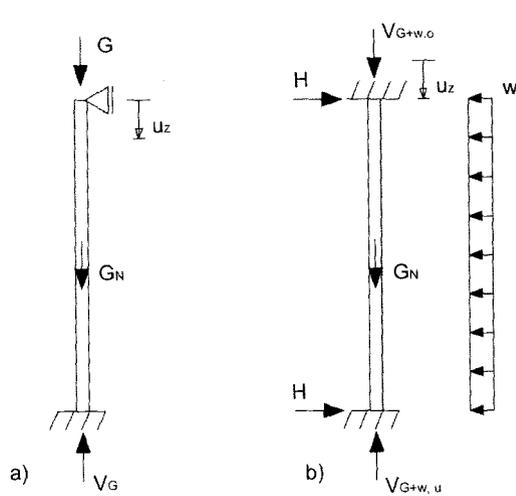


Figure 12

Resulting forces for

- a) self weight $G_N = 1.77$ kN, $G_1 = 5.84$ kN, $V_G = 7.61$ kN,
 b) self weight plus wind load $G_N = 1.77$ kN, $V_{G+w,u} = 10.25$ kN,
 $V_{G+w,0} = 6.71$ kN

greater ($\sigma_{\max} = 10.0$ MN/m²). The reason for greater stress under a smaller load is a deeper opening of the joints. This causes a great edge pressure.

The results depend very much on the value of the modulus of elasticity. A great value leads to a stiff structure, small displacements and small compressive stress due to a small excentricity of the thrust line.

The horizontal beam

The horizontal beam acts as a horizontal support of the bars. It has to work like the bars. However there is a big difference. The beam is not pre-stressed by self weight. Only a flat arch inside the cross section and between rigid horizontal supports can take the load. The results of the analysis show rather small displacements and acceptable compression stress. However the resultant forces at the horizontal supports are about 30 kN.

Figure 14

Modelling of the horizontal beam (displacement 50-times magnified)

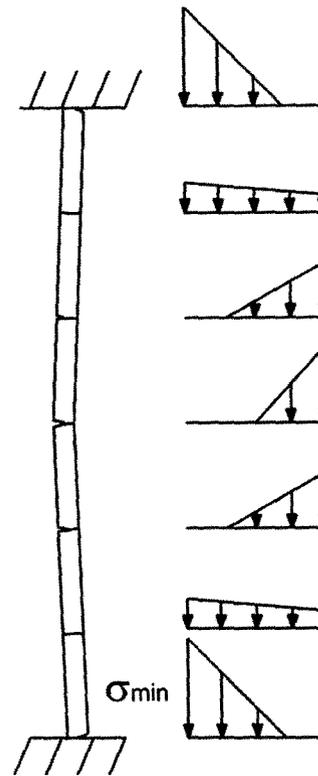


Figure 13

Compressive stress at the joints under self weight + wind load: Maximum compressive stress $\sigma_{\min} = 7.9$ MN/m², maximum displacement in horizontal direction $u_b \sim 0.5$ cm

CONCLUSION

The theoretical model of a framework between rigid boundaries leads to reasonable results. Not only the straight bars but also the tracery at the top of the window will behave like that. At the tracery the flow of forces is more complicated. However it is very stiff in relation to the bars and allows different load paths. In figure 15 the most important ribs are marked. The hanging arch gives an additional stiffness in vertical direction.

In reality the ironed saddle bars will participate to a certain extend. At the described window showing a width of 3.7 metres the saddle bars take only a little part of the wind load. Nevertheless the saddle bars are very important. At the first place they provide

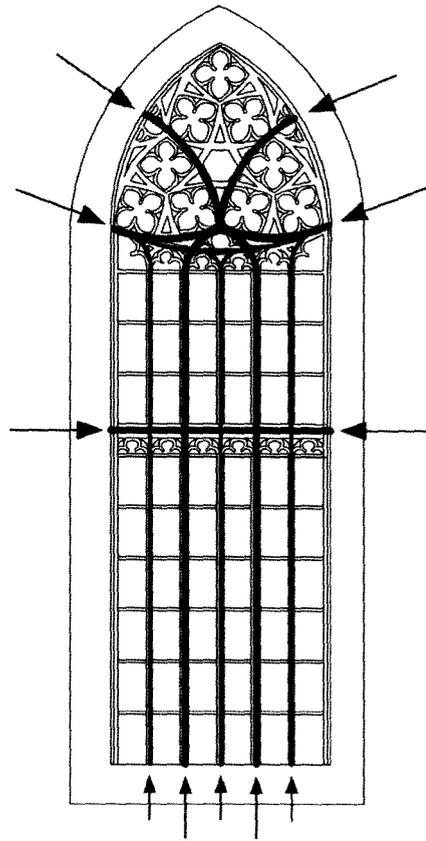


Figure 15
Load path and reaction forces

horizontal stiffness in two directions during construction. When the window is finished the saddle bars are essential for the stability of the bars in the plane of the window. In addition they provide a

second load bearing system in case of failure of a stone element or even of the total stone skeleton due to extraordinary loads. In such a case great deformations will occur but a total collapse may be avoided.

The stability of the tracery window is proved due to the fact of its age of about five hundred years. The main objective of the analysis is to develop a guideline for an appropriate repair concept of the window. The most important conclusion derived from the analysis is to provide rigid boundaries and stiff connections between the stones. During the last five hundred years the overall movements of the masonry of the choir lead to a small but significant increase of the width of the window. The described arch action is reduced due to cracks and gaps between the stone especially at the horizontal beam. Repairing the framework of the window at the first place stiff connections between the stone elements have to be restored. In addition the masonry of the choir has to be assessed thoroughly. It has to be ensured that the masonry is not subjected to a horizontal thrust of the roof structure. The horizontal thrust of the vaults in relation to the stiffness of the masonry has to be examined. It has to be discussed whether a horizontal tie rod at the level of the horizontal beam should be inserted in order to pre-stress the beam or at least to take the forces.

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